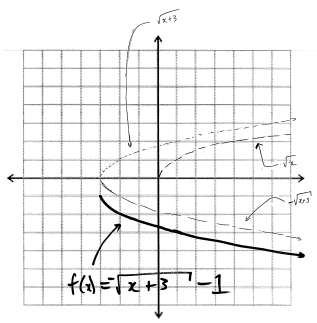
Precalculus-04, Test 2 Review Answers

Dr. Graham-Squire, Fall 2013

1. Starting with the graph of $y = \sqrt{x}$, shift, flip and/or stretch the graph to find the graph of $y = -\sqrt{x+3} - 1$.



2. Let $f(x) = \dfrac{1}{\sqrt{x}}$, $g(x) = x^3$, and $h(x) = x^2 + 2x + 3$. Find the composition of functions $g(h(f(x)))$. Simplify your answer if possible.

   Ans: $\left(\dfrac{1}{x} + \dfrac{2}{\sqrt{x}} + 3\right)^3$ or $\frac{1}{x^3} + \frac{6}{x^{2.5}} + \frac{9}{x^2} + \frac{26}{x^{1.5}} + \frac{63}{x} + \frac{54}{\sqrt{x}} + 27$

3. Graph each function to confirm that it is one-to-one. If it is not, restrict the domain so that it is one-to-one. Then find the inverse for each function.

   (a) $f(x) = \dfrac{2-3x}{4+x}$

   Ans: One-to-one. The inverse is $f^{-1}(x) = \dfrac{2-4x}{x+3}$

   (b) $f(x) = \dfrac{1}{2}(x-7)^2 + 3$

   Ans: Not one-to-one. Two answers- if the domain is restricted to $(-\infty, 7]$, then the inverse function is $f^{-1}(x) = -\sqrt{2(x-3)} + 7$. If the domain is restricted to $[7, \infty)$, then the inverse function is $f^{-1}(x) = \sqrt{2(x-3)} + 7$.

4. The owner of a luxury motor yacht that sails among the Greek islands charges \$600/person if exactly 20 people sign up for the cruise (which gives a total revenue of $600 \cdot 20 = 12,000$ dollars) . However, if more than 20 people sign up (up to the maximum capacity of 90) for the cruise, then each fare is reduced by \$4 for each additional passenger. Thus if there are 21 people, the fare is \$596 per person for everyone, for a total revenue of $596 \cdot 21 = 12516$ dollars. If there are 22 people then the fare is \$592 per person, etc. Assuming at least 20 people sign up for the cruise, let x be the number of passengers over 20 who sign up for the cruise. Answer the following questions:

   (a) Construct a quadratic function $r(x)$ to represent the total revenue in terms of x. **Ans**:$r(x) = (600 - 4x)(x + 20)$. The 600-4$x$ is the price per passenger, and the $x + 20$ is the total number of passengers.

   (b) Find the number of passengers that would give the *maximum* amount of revenue for the owner of the yacht. **Ans**: Multiplying out $r(x)$ gives $r(x) = -4x^2 + 520x + 12000$. Then $x = \frac{-b}{2a} = -520/-8 = 65$, so the number of passengers who would give a maximum revenue is 65+20=85.

(c) What is the maximum revenue possible? **Ans**: $r(65) = 340 \cdot 85 = 28,900$ dollars.

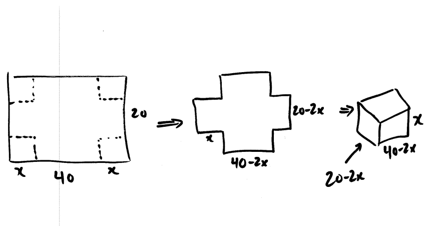
(d) Explain how you could find (b) and (c) using a graph of $r(x)$, and how you would find them without using a graph. **Ans**: Without using a graph it is how I did it above, finding -b/2a and then substituting back into the equation. From the graph, you would find the local maximum point on the graph. The x-value is the answer for (b), the y-value is the answer for (c).

5. Find the quotient and remainder for the division $\dfrac{x^5 - 2x^4 + x^3 - 3x + 1}{x^2 + 4x - 1}$

   Ans: Quotient is $x^3 - 6x^2 + 26x - 110$, remainder is $463x - 109$.

6. An open box with a volume of 1500 cm^3 is to be constructed by taking a piece of cardboard 20 cm by 40 cm, cutting squares of length x from each corner, and folding up the sides. Show that this can be done in two different ways and find the exact value of x in each case. Note that "exact" means that I do not just want a decimal approximation!

   (a) Draw a diagram of the situation. Start with a 20×40 rectangle, then show how the corners are cut out, then show how it is folded up into a box. **Ans**:



   (b) Write an equation (in terms of x) that represents the volume of box, and then move stuff to one side so that you have a polynomial equal to zero. **Ans**: Volume= $l \cdot w \cdot h = (40 - 2x)(20 - 2x)(x)$. Since the volume is 1500, we have the equation

$$1500 = (40 - 2x)(20 - 2x)(x) \text{ or } 0 = 4x^3 - 120x^2 + 800x - 1500$$

   (c) Use graphing and/or factoring techniques to find all the zeroes of the polynomial from part (b). **Ans**: Graphing $f(x) = 4x^3 - 120x^2 + 800x - 1500$, it looks like $x = 5$ is a zero. Polynomial long division gives $f(x) = (x-5)(4x^2 - 100x + 300)$. Using the quadratic formula on $4x^2 - 100x + 300$ gives zeroes of $(100 \pm \sqrt{5200})/8$. One of those roots is too large, though, because $x = (100 + \sqrt{5200})/8 \approx 21.5$. Our width of the rectangle is only 20, so x cannot be bigger than 10 and so we toss out that zero as a possible solution. Thus the two possible answers are $x = 5$ and $x = (100 - \sqrt{5200})/8$

7. Find all zeroes for the following polynomials, both real and complex:

   (a) $x^5 + 5x^3 - 36x$

   Ans: $x^5 + 5x^3 - 36x = x(x^4 + 5x^2 - 36) = x(x^2 + 9)(x^2 - 4) = x(x + 3i)(x - 3i)(x + 2)(x - 2)$ Zeroes are 0, $\pm 3i$, ± 2.

   (b) $x^5 - 2x^4 + x^3 - 8x^2 + 16x - 8$ (Hint: try factoring by grouping- You will have to group in a different manner than previous problems, though, possibly rearranging some of the terms).

   Ans: $x^5 - 2x^4 + x^3 - 8x^2 + 16x - 8 = (x^5 - 2x^4 + x^3) - 8x^2 + 16x - 8 = x^3(x^2 - 2x + 1) - 8(x^2 - 2x + 1) = (x^3 - 8)(x^2 - 2x + 1) = (x^3 - 8)(x - 1)^2 = (x - 2)(x^2 + 2x + 4)(x - 1)^2$. Using quadratic formula on $x^2 + 2x + 4$ gives zeroes=2, 1, and $-1 \pm i\sqrt{3}$.

8. (a) Find all x and y intercepts and all vertical and horizontal asymptotes for the rational function

$$f(x) = \frac{-2(x-3)(x+3)}{x(x+5)(x-2)}$$

Ans: x-intercepts at $x = 3$ and $x = -3$, y-intercept does not exist. Vertical asymptotes at $x = 0$, $x = -5$ and $x = 2$; Horizontal asymptote at $y = 0$.

(b)Sketch the graph of $f(x)$.

Ans: You need to look at how the function acts as x approaches the asymptotes to know where the graph goes. For example, as x approaches 2 from the right the numerator will be $- \cdot - \cdot +$ (which is positive) and the bottom will be $+ \cdot + \cdot +$ which is positive, so the graph is going to positive infinity as x approaches 2 from the left.